Equation of state of hyperonic nuclear matter at zero and finite temperatures with the variational method

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Outline

1: Introduction
2: Hyperon EOS at zero temperature
3: Hyperon EOS at finite temperature
4: Summary
1. Introduction

Hyperon interactions play an important role in compact astrophysical objects.

Neutron star

Structure is governed by the nuclear equation of state (EOS) at zero temperature.

**HYPERON PUZZLE**

- EOS becomes softer due to the hyperon mixing.
- Maximum mass of neutron star tends to be lower than the observational data.

The hyperon mixing in neutron stars has been studied with various nuclear theories.

- Relativistic mean field theory (S. Weissenborn et al., PRC 85 (2012) 065802)
- Brueckner-Hatree-Fock theory (H. Schulze, T. Rijken, PRC 84 (2011) 035801)
- Variational many-body theory (H. Togashi et al., accepted in PRC)
### Nuclear EOS for core-collapse simulations

*Nuclear EOS at finite temperature → Core-collapse supernova (SN)*

1. **Lattimer-Swesty EOS**: *The Skyrme-type interaction*  (NPA 535 (1991) 331)
2. **Shen EOS**: *The Relativistic Mean Field Theory*  (NPA 637 (1998) 435)

<table>
<thead>
<tr>
<th>Nuclear Interaction</th>
<th>$n_{\text{sat}}$ (fm$^{-3}$)</th>
<th>$BE/A$ (MeV)</th>
<th>$K$ (MeV)</th>
<th>$Q_{\text{int}}$ (MeV)</th>
<th>$J$ (MeV)</th>
<th>$L$ (MeV)</th>
<th>type of int. used in</th>
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<tbody>
<tr>
<td>SKa</td>
<td>0.155</td>
<td>16.0</td>
<td>263</td>
<td>-300</td>
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<td>176</td>
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<td>272</td>
<td>203</td>
<td>37.3</td>
<td>118.2</td>
<td>RMF, HS(NL3)</td>
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<tr>
<td>FSUgold</td>
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<td>16.3</td>
<td>230</td>
<td>-524</td>
<td>32.6</td>
<td>60.5</td>
<td>RMF, SH(FSU1.7), HS(FSUgold)</td>
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<td>FSUgold2.1</td>
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<td>16.3</td>
<td>230</td>
<td>-524</td>
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<td>60.5</td>
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<td>-290</td>
<td>31.3</td>
<td>47.2</td>
<td>RMF, HS(IUFSU)</td>
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<tr>
<td>DD2</td>
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<td>16.0</td>
<td>243</td>
<td>169</td>
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<td>RMF, HS-DD2, BHBA, BHBA φ</td>
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<td>-285</td>
<td>36.9</td>
<td>110.8</td>
<td>RMF</td>
</tr>
</tbody>
</table>

There are no SN EOSs based on the microscopic many-body theory.
SN-EOS based on the microscopic many-body theory

We have constructed the nuclear EOS for core-collapse simulations with the variational method.

Collaboration with M. Takano (Waseda University), K. Sumiyoshi (Numazu College of Tech.), Y. Takehara, S. Yamamuro, K. Nakazato, H. Suzuki (Tokyo Univ. of Science)

**Method** : Cluster variational method

**Potential** : AV18 + UIX


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**Energies of uniform matter**

**Application to Neutron Star**

**Application to Supernova simulation**

We extend our variational method to consider additional contributions from Λ hyperons.
2. Hyperon EOS at zero temperature

Two-body Hamiltonian

\[ H_2 = -\sum_{i} \left[ m_i c^2 + \frac{\hbar^2}{2m_i} \nabla_i^2 \right] + \sum_{i<j} V_{ij} \]

Three-body Hamiltonian

\[ H_3 = \sum_{i<j<k} V_{ijk} \]

- **NN potential**: AV18 two-nucleon potential (PRC 51 (1995) 38)

- **ΛN and ΛΛ potentials**: Central potentials
  - The \textit{ab initio} variational calculations for Λ hypernuclei reproduce their experimental eigenvalues.

**The experimental data of hypernuclei give no information on the odd-state part of the ΛΛ interactions.**
We investigate the effects of on the compact astrophysical objects.

The odd-state part of the $\Lambda\Lambda$ interaction

We prepare four different models for the odd-state part of the $\Lambda\Lambda$ interaction.

Type 1: *The most attractive*
Type 2: *Less attractive*
Type 3: *Slightly repulsive*
Type 4: *The most repulsive*

The repulsion strength of Type 4 is comparable to that of the odd-state repulsion of $\Lambda N$ interaction.

The repulsive effect increases monotonically from Type 1 to Type 4.

- We investigate the effects of the odd-state part of bare $\Lambda\Lambda$ interactions on the compact astrophysical objects.
Energy of hyperonic nuclear matter

Energy per baryon

\[ E(n_n, n_p, n_\Lambda) = E_2(n_n, n_p, n_\Lambda) + E_3^N \]

\( E_2 \): The expectation value of \( H_2 \) with the Jastrow wave function in the two-body cluster approximation.

\[ \Psi = \text{Sym} \left[ \prod_{i<j} f_{ij} \right] \Phi_F \]

\( \Phi_F \): Fermi-gas wave function

\( E_3^N \): Three-nucleon energy

Based on the expectation value of \( H_3 \) with the Fermi-gas wave function

\[ E_3^N = \langle \alpha H_3^R + \beta H_3^{2\pi} \rangle_F + E_{\text{corr}} \]

(NPA902 (2013) 53)
Mass-radius relations of neutron stars

Maximum mass of neutron stars

<table>
<thead>
<tr>
<th>Type</th>
<th>Mass (M⊙)</th>
</tr>
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<tbody>
<tr>
<td>Type 1</td>
<td>1.52 M⊙</td>
</tr>
<tr>
<td>Type 2</td>
<td>1.60 M⊙</td>
</tr>
<tr>
<td>Type 3</td>
<td>1.65 M⊙</td>
</tr>
<tr>
<td>Type 4</td>
<td>1.73 M⊙</td>
</tr>
<tr>
<td>without Y</td>
<td>2.22 M⊙</td>
</tr>
</tbody>
</table>

The maximum mass increases. (1.52 M⊙ → 1.73 M⊙)

Application to Neutron Star

J0348+0432: Science 340 (2013) 1233232

Shaded region is the observationally suggested region by Steiner et al. (Astrophys. J. 722 (2010) 33)
3. Hyperon EOS at finite temperature

Free energy $F$ is expressed by the average occupation probabilities.

The average occupation probability

$$f_i(k) = \left\{ 1 + \exp\left[ \frac{\varepsilon_i(k) - \mu_{0i}}{k_B T} \right] \right\}^{-1}$$

$\varepsilon_i(k)$: Single particle energy

$$\varepsilon_i(k) = \frac{\hbar^2 k^2}{2m_i^*} \quad (i = p, n, \Lambda)$$

$m_i^*$: Effective mass of baryons

Free energies are minimized with respect to $m_i^*$

Free energy of hyperonic nuclear matter (Type 4)
Application to supernova matter

We calculate the onset density of $\Lambda$ hyperons in hot dense matter with the equilibrium condition $\mu_n = \mu_{\Lambda}$.
4. Summary

We construct the EOS of nuclear matter including $\Lambda$ hyperons at zero and finite temperatures by the variational method.

- The obtained thermodynamic quantities are reasonable.
- The repulsion in the odd-state $\Lambda\Lambda$ interaction raises the maximum mass of neutron star. ($1.52 \, M_\odot \rightarrow 1.73 \, M_\odot$)
- The onset density of $\Lambda$ is insensitive to the odd-state $\Lambda\Lambda$ interaction.
- TBF shifts the onset density of $\Lambda$ to the higher density region at low temperatures.

Future Plans

- Construction of the EOS table for core-collapse simulations
- Taking into account mixing of other hyperons ($\Sigma^-, \Sigma^0, \Sigma^+, \Xi^0, \Xi^-$)
- Employing more sophisticated baryon interactions (e.g. Nijmegen)